A COUPLED MULTIDOMAIN SPACE-TIME FINITE ELEMENT METHOD TO SOLVE THE NONLINEAR ONE-DIMENSIONAL EQUATIONS OF BLOOD FLOW IN ELASTIC VESSELS

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Flow and pressure waves, originating due to the contraction of the heart, propagate along the deformable vessels and reflect at branching points or other discontinuities. The size and complexity of the cardiovascular system necessitate a “multiscale” approach including “upstream” regions of interest (large arteries) coupled to reduced-order models of “downstream” vessels [1,2]. Previous efforts to couple upstream and downstream domains have included specifying resistance [3] and impedance [4] outflow boundary conditions for the non-linear one-dimensional wave propagation equations, and iterative coupling between three-dimensional and one-dimensional numerical methods [5]. We have developed a new approach to solve the one-dimensional non-linear equations of blood flow in elastic vessels utilizing a space-time finite element method with GLS-stabilization for the upstream domain [3], and a boundary term to couple to the downstream domain:

$$\textit{Find } \boldsymbol{U} \textit{ such that for all weighting functions } \boldsymbol{W} \left(\textit{with } \boldsymbol{F}, \boldsymbol{K}, \boldsymbol{G} \textit{ and } \boldsymbol{\mathcal{L}} \textit{ derived from the strong form}\right) \textit{ on the domain } (0,B)\times(0,T)$$

$$\int_0^T\int_0^B \left(\boldsymbol{W}_{,t}^{\boldsymbol{T}}\boldsymbol{U} + \boldsymbol{W}_{,z}^{\boldsymbol{T}}\left(\boldsymbol{F}(\boldsymbol{U}) - \boldsymbol{K}\boldsymbol{U}_{,z}\right) + \boldsymbol{W}^{\boldsymbol{T}}\boldsymbol{G}(\boldsymbol{U})\right)dz\,dt - \int_0^B \left[\boldsymbol{W}^{\boldsymbol{T}}(z,t)\ \boldsymbol{U}(z,t)\right]_{t=0}^{t=T} dz + \int_0^T \left[\boldsymbol{W}^{\boldsymbol{T}}\left(\boldsymbol{F}(\boldsymbol{U}) - \boldsymbol{K}\boldsymbol{U}_{,z}\right)\right]_{z=0} dt$$
$$- \boxed{\int_0^T \left[\boldsymbol{W}^{\boldsymbol{T}}\left(\boldsymbol{M}(\boldsymbol{U}) + \boldsymbol{H}\right)\right]_{z=B} dt} + \int_0^T\int_0^B \left(\mathcal{L}(\boldsymbol{U})^T \boldsymbol{W}\right)^T \boldsymbol{\tau}\ \left(\mathcal{L}(\boldsymbol{U})\boldsymbol{U}\right) dz\,dt = 0$$

The operators $\boldsymbol{M}$ and $\boldsymbol{H}$ in the outflow boundary condition above are derived following an approach analogous to the DtN method [6]: in the downstream domain, we solve simplified 0D/1D equations to derive relationships between pressure and flow accommodating periodic and transient phenomena with a consistent formulation for different boundary condition types. Clinical problems motivating the present work, the methods employed, results obtained, and future directions will be presented.

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